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Norwegian University of Science and Technology

Cryptography 101 & Secure Computation

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Overview

- Cryptography 101
  - Symmetric-Key Encryption
  - Public-Key Encryption
  - Hash Functions
  - Digital Signatures
- Secure Computation
  - Secret Sharing
  - Multi-Party Computation
  - Fully Homomorphic Encryption
  - Secure computation in practice
- Cryptography & Machine Learning

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Cryptography 101

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Symmetric-Key Encryption I

A symmetric-key encryption-scheme is an encryption-system for two parties with two functions $\texttt{Enc}(\cdot,\cdot)$ and $\texttt{Dec}(\cdot,\cdot)$ together with a shared key k:

- $\texttt{Enc}$ takes as input a key k and a message m, and outputs a ciphertext c: $\texttt{Enc}(k, m) = c$.
- $\texttt{Dec}$ takes as input a key k and a ciphertext c, and outputs a message m: $\texttt{Dec}(k, c) = m$.

Only the two parties that know the key can encrypt and decrypt messages.

Symmetric-Key Encryption II

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Symmetric-Key Encryption III

Properties of a symmetric-key encryption-scheme:

- Both parties need to know the key k in advance
- Usually have keys and blocks of size 128 or 256 bits
- Super fast (only using XOR, AND, vectors etc.)
- Length of ciphertext $\approx$ Length of plaintext

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Symmetric-Key Encryption IV

Security of symmetric-key encryption:

- The best way to break it is to guess the key
- It does not hide the length of the encrypted data
- Might be vulnerable to side-channel attacks
- Might be vulnerable to attacks by quantum computers

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Public-Key Encryption I

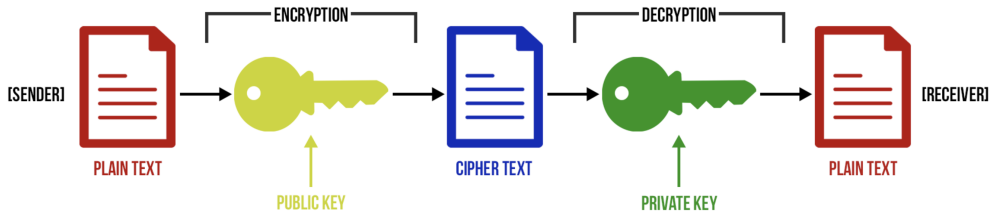
A public-key encryption-scheme is an encryption-system for two parties with two functions $\texttt{Enc}(\cdot,\cdot)$ and $\texttt{Dec}(\cdot,\cdot)$ together with a public key k_p and a secret key k_s:

— $\texttt{Enc}$ takes as input a public key k_p and a message m,
and outputs a ciphertext c: $\texttt{Enc}(k_p, m) = c$.

— $\texttt{Dec}$ takes as input a secret key k_s and a ciphertext c,
and outputs a message m: $\texttt{Dec}(k_s, c) = m$.

Everyone can encrypt a message, but only one party can decrypt.

Public-Key Encryption II



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Public-Key Encryption III

Properties of a public-key encryption-scheme:

- Only one party knows the private key, everyone knows the public key
- Pretty slow, working with big numbers thousands of bits long
- Length of ciphertext $>>$ Length of plaintext

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Public-Key Encryption IV

The RSA cryptosystem:

- Choose two prime numbers p and q and let $n = p \cdot q$
- Choose a number e and find a special number d depending on p and q
- The public key is the two numbers (e, n)
- The private key is the two numbers (d, n)

Public-Key Encryption V

The RSA cryptosystem:

- Encryption of message m: $\texttt{Enc}((e, n), m) = m^e \mod n = c$
- Decryption of ciphertext c: $\texttt{Dec}((d, n), c) = c^d \mod n = m$

Public-Key Encryption VI

Security of RSA public-key encryption:

- The best way to break RSA is to factorize n into p and q
- The cryptosystem must be randomized to be secure
- Might be vulnerable to padding attacks
- Might be vulnerable to side-channel attacks
- Will be broken by attacks by quantum computers

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Hash Functions I

A hash function `H` is a deterministic function that takes input of arbitrary length, and produce a fixed length output. A hash function has the three following properties:

- Collision resistance,
- Pre-image resistance,
- Second pre-image resistance.

Hash Functions II

Collision resistance:
It should be difficult to find two different messages m_1 and m_2 such that $\mathtt{H}(m_1) = \mathtt{H}(m_2)$.

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Hash Functions III

Pre-image resistance:
Given a hash value h it should be difficult to find any message m such that $h = \mathtt{H}(m)$.

Hash Functions IV

Second pre-image resistance:
Given an input m_1, it should be difficult to find a different input m_2 such that $\mathtt{H}(m_1) = \mathtt{H}(m_2)$.

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Hash Functions V

Security of hash-functions:

- The best way to break a hash-function is to guess inputs
- The best way to get collisions is the birthday attack
- Commonly used hash-functions have outputs of length 256 and 512 bits
- Might be vulnerable to attacks by quantum computers

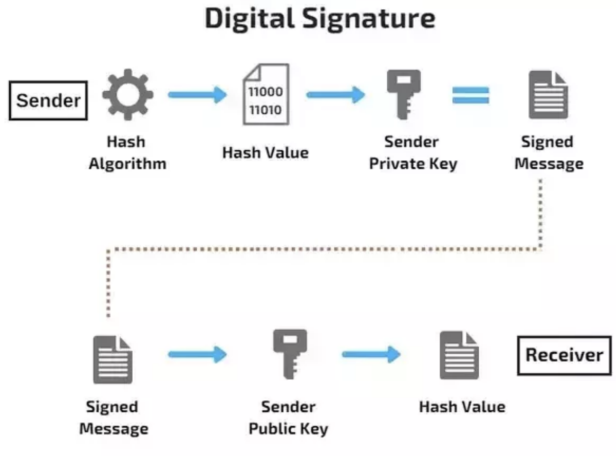
Digital Signatures I

A digital signature-scheme is a signature-system for two parties with two functions $\texttt{Sign}(\cdot,\cdot)$ and $\texttt{Verify}(\cdot,\cdot,\cdot)$ together with hash function $\texttt{H}$, a secret key k_s and a public key k_p:

- $\texttt{Sign}$ takes as input a secret key k_s and a message m, and outputs a signature σ: $\texttt{Sign}(k_s, m) = \sigma$.
- $\texttt{Verify}$ takes as input a public key k_p, a signature σ and a message m, and outputs a message **True** or **False**: $\texttt{Verify}(k_p, \sigma, m) =$ **True** or **False**.

Only one party can sign a message, but everyone can verify a signature.

Digital Signatures II



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Digital Signatures III

The RSA digital signature-system:

- Choose two prime numbers p and q and let $n = p \cdot q$
- Choose a number v and find a special number s depending on p and q
- The public key is the two numbers (v, n)
- The private key is the two numbers (s, n)

Digital Signatures IV

The RSA digital signature-system:

— Signature of message m: $\texttt{Sign}((s, n), m) = (\texttt{H}(m))^s \mod n = \sigma$

— Verification of signature σ: $\texttt{Verify}((v, n), \sigma, m) = \sigma^v \mod n \stackrel{?}{=} \texttt{H}(m)$

Digital Signatures V

Scott Hanselman
@shanselman

Follow

HTTPS & SSL doesn't mean "trust this." It means "this is private." You may be having a private conversation with Satan.

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Secure Computation

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Secure Computation

Goal:

being able to do computations on encrypted data, and therefore compute valuable information without violating privacy.

Secret Sharing I

Goal:

share a secret with a group in a way so that none of the people can reconstruct the secret on their own, but some threshold of members can reconstruct the secret if they work together.

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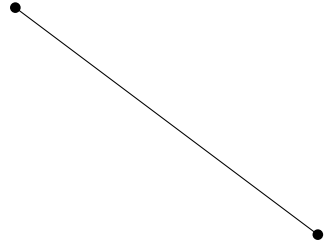
Secret Sharing II

First; some geometry:

- How many points in the plane do you need to define a straight line?
- How many points in the plane do you need to define a parabola?

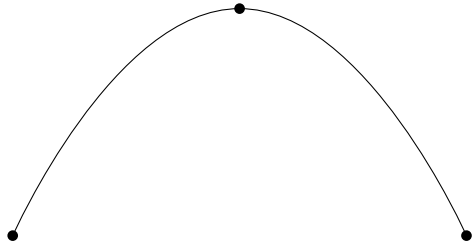
Secret Sharing III

$y = ax + b$



Secret Sharing IV

$$y = ax^2 + bx + c$$



Secret Sharing V

We have a secret number, say, $b = 4$.

We want to share this secret with three friends in a way so that none of them knows our secret, but if at least two of them work together, then they are able to reconstruct the secret.

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Secret Sharing VI

We want to construct a polynomial of degree one: $p(x) = ax + b$,
and give one point $p(1), p(2), p(3)$ to each of our three friends.

Assume that we can only chose coefficient from the numbers 0 to 4.
Our secret number will be $b = 4$, and we randomly choose, say, $a = 2$.

Secret Sharing VII

Then $p(0) = 4$ is the secret we want to share.

We share the following with our friends:

- $p(1) = 2 \cdot 1 + 4 = 6 \mod 5 = 1$ with player A,
- $p(2) = 2 \cdot 2 + 4 = 8 \mod 5 = 3$ with player B,
- $p(3) = 2 \cdot 3 + 4 = 10 \mod 5 = 0$ with player C.

Secret Sharing VIII

Individually,

- player A has the value $p(1) = 1$,
- player B has the value $p(2) = 3$,
- player C has the value $p(3) = 0$,

but none of them know what the secret is, because they need two points to be able to reconstruct the whole line.

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Multi-Party Computation I

Goal:

allow a group to compute a public function,
without having to reveal their individual input.

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Multi-Party Computation II

Procedure:

- Everyone secret-share their input with everyone
- Addition and scalar multiplication is done locally
- Multiplications require sending a new secret-share to every party

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Multi-Party Computation III

Security:

- Secret sharing with threshold $n - 1$ of n parties
- Parties can provide "false" input
- Can notice and abort if someone is cheating

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Multi-Party Computation IV

Advantages:

- Computation is really fast
- Post-quantum security

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Multi-Party Computation V

Disadvantages:

- Require a lot of communication
- Everyone must be "online" at all time

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Fully Homomorphic Encryption I

Goal:

allow a third party to compute a function,
without knowing the input nor the output.

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Fully Homomorphic Encryption II

"Noisy encryption":

- The encryption of a message contains some extra "noise"
- As long as the "noise" is small, you can still decrypt
- Adding ciphertexts also adds the "noise"
- Multiplying ciphertexts also multiplies the "noise"
- When the "noise" gets larger, you can "bootstrap" to make it smaller

Fully Homomorphic Encryption III

Procedure:

- The user encrypts their data and sends it over
- The third party does the computations on the encrypted data
- The user gets the encrypted data back and then decrypts

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Fully Homomorphic Encryption IV

Advantages:

- No communication while computing
- Small functions are fast to compute
- Post-quantum security

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Fully Homomorphic Encryption V

Disadvantages:

- Require a lot of computation
- "Bootstrapping" is expensive

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Secure computation in practice I

Multi-Party Computation:

- Pre-computation independent on data and function using FHE
- Secret share information as normal
- Fast MPC using the pre-computed data

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Secure computation in practice II

Fully Homomorphic Encryption:

- Agree on function before choosing parameters
- Choose parameters large enough to handle some "noise"
- Do as little "bootstrapping" as possible

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Secure computation in practice III

Some applications in use:

- Journalist can share sensitive information in case anything happens...
- Private companies can analyze medical information without violating privacy
- Competing companies can work together without sharing data

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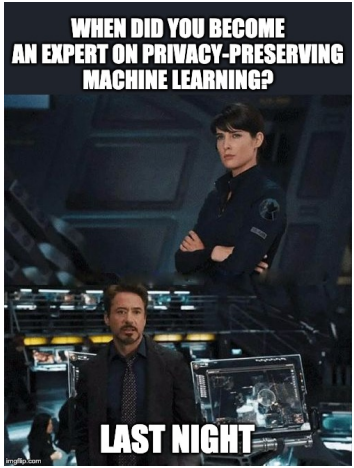
Cryptography & Machine Learning

Cryptography & Machine Learning I



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Cryptography & Machine Learning II

Cryptography for machine learning:

- Training neural networks on encrypted data
- Prove correctness of machine learning algorithms
- Protect machine learning algorithms against abuse
- Training neural networks while keeping the training model secret

Cryptography & Machine Learning III

Machine learning for security:

- Detection of vulnerabilities and attacks
- Crypto- and malware-analysis
- Machine learning for attacking cryptography

Cryptography & Machine Learning IV

Resources:

— Rivest, Asiacrypt 1991. “Cryptography and machine learning”.
- https://people.csail.mit.edu/rivest/pubs/Riv91.pdf

— Goldwasser, Crypto 2018. "From Idea to Impact, the Crypto Story"
- https://www.youtube.com/watch?v=culuNbMPP0k

— Alani, 2019. "Applications of Machine Learning in Cryptography: A Survey."
- https://dl.acm.org/citation.cfm?doid=3309074.3309092

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Thank You! Questions?

Email: tjerand.silde@ntnu.no
Talk: `www.tjerandsilde.org/talks`